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# Mie and Debye scattering in dusty plasmas

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We calculate the total field scattered by a charged sphere immersed in a plasma using a unified treatment that includes the usual Mie scattering and the scattering by the Debye cloud around the particle. This is accomplished by use of the Dyadic Green function to determine the field radiated by the electrons of the Debye cloud, which is then obtained as a series of spherical vector wave functions similar to that of the Mie field. Thus we treat the Debye-Mie field as a whole and study its properties. The main results of this study are (1) the Mie (Debye) field dominates at small (large) wavelengths and in the Rayleigh limit the Debye field is constant; (2) the total cross section has an interference term between the Debye and Mie fields, important in some regimes; (3) this term is negative for negative charge of the grain, implying a total cross section smaller than previously thought; (4) a method is proposed to determine the charge of the grain (divided by a certain suppression factor) and the Debye length of the plasma; (5) a correction to the dispersion relation of an electromagnetic wave propagating in a plasma is derived.

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## I. INTRODUCTION

In this work we study the scattering of an electromagnetic wave by the system composed of a charged dust grain, assumed spherical, and the surrounding screening Debye sphere. This problem arises in the context of the study of dusty plasmas, but is also related closely to the theory of Mie scattering. Therefore, this is a classical electromagnetism problem applied to the environment of laboratory as well as natural plasmas, such as ionospheric or space plasmas.

The study of dusty plasmas has been growing in importance in the last years [1–12]. Roughly, a dusty plasma is composed of dust particulates surrounded by an electron-ion plasma. Due to the electron and ion currents, photoemission and secondary emission, the dusts become charged up to hundreds or thousands of elementary charges [5]. The dusty plasmas are present almost everywhere: interstellar clouds, circumstellar and protoplanetary accretion disks, planetary rings, comets, planetary magnetospheres [4,6,7], and in laboratory environments, such as plasma processing devices [8] or, more recently, in the promising new field of dusty crystals [9,10].

Mie theory is a classical area of optical physics [13,14]. It describes the scattering of electromagnetic waves by a spherical particle and it has been successfully applied to a vast range of problems, e.g., in the study of atmospheric aerosols or rainfall measurements [15].

Mie theory is also used in a variety of plasma environments, for example, in the study of noctilucent clouds [16], for sizing contaminants in plasma processing chambers [11,12] or in the study of scattered light by intergalactic dust [1,17]. However, the effect of the plasma is not taken into account in these applications. In this paper we show some situations where scattering by the plasma can be as important as Mie scattering.

The physical picture of our problem is simple: the highly charged dust is screened by the Debye cloud that forms around it. This cloud is a heterogeneity in the electron and ion average densities and thus it will scatter the incident radiation. Therefore, there is the simultaneous scattering by the spherical dust [Mie scattering (MS)], and by the Debye cloud [Debye scattering, (DS)].

The importance of the DS can be easily understood considering a limiting case. If the incident electromagnetic radiation has wavelength $\lambda \gg \lambda_D$ (where $\lambda_D$ is the Debye length), the charges inside the Debye sphere scatter radiation coherently and thus the scattering cross section $\sigma_D$ for this process is strongly enhanced relative to the cross section of a single electron, $\sigma_0$ (the Thomson cross section), being approximately given by $Z^2\sigma_0$, where $Z \sim 10^4,10^5$ is the charge number of the grain.

This argument was first presented by Tsytovich *et al.* [18] and Bingham *et al.* [19], who have first studied the problem of DS starting from the Vlasov equation (including an equilibrium distribution function for the electrons around the dust) and Fourier transforming the wave equation. La Hoz [20] developed the theory for the study of radar bckscattering from dusty plasmas. Following the same lines Vladimirov [21] included the effect of dust charge fluctuations [22] in the process, but using essentially the same methods. He found a dependence of DS on the parameters characterizing the dust charging.

In this work we treat the problem of DS following a method closely related to the method used to study MS, expressing the Debye field in a Mie-like series of spherical vector wave functions [23,24]. The basic idea is to solve the

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wave equation with the aid of the Dyadic Green Function [25,24] (DGF), instead of going to the $k$ space. The DGF is expressed in a series of spherical vector wave functions, and thus the Debye field is determined in a similar form.

The advantages of this approach are the following: ease in the calculations of cross section, irradiances, and polarization; direct physical interpretation, in parallel to MS; possibility of describing the Mie-Debye process as a continuum, being dominated by MS (DS) in the short (long) wavelength regime, and having an intermediate regime where the Debye field and the Mie field interfere to produce a mixed cross section: $\sigma_{TOT}=\sigma_M+\sigma_D+\sigma_{MD}$. Furthermore, we shall see that in the forward and backward directions the expansion series for DS can be summed exactly, giving as a bonus a method for determining the charge (divided by a certain correction factor) and the Debye length. Finally, the relation between forward amplitude scattering and dielectric constant allows the determination of a correction to the dispersion relation of a electromagnetic wave propagating in a dusty plasma.

Finally, it should be stressed that we treat a static problem, that is, the Debye cloud is assumed to be a static perturbation around the charged sphere. Also, we treat mainly the problem of one scatterer. However, if the grains are far enough apart so that their interaction is negligible, if they are randomly distributed, and if they have all the same radius, then we can assume that the total irradiance is the sum of irradiances.

## II. DESCRIPTION AND FORMAL SOLUTION TO THE PROBLEM

### A. Equation for the Debye field derived from Maxwell and fluid equations for the plasma

Consider a plasma consisting of electrons, ions, and dust grains, positively or negatively charged. In equilibrium, the spatially averaged densities of these components are related by

$$n_{eq,e}-n_{eq,i}+Zn_d=0, \tag{2.1}$$

where $n_{eq,e}$ and $n_{eq,i}$ are the electron and ion equilibrium average densities (we have assumed unitary charge for the ions), $n_d$ is the density of grains, and $Z$ the number of charges attached to it, which can be positive or negative. The Debye shielding cloud formed around the grains represents a (stationary) local perturbation in the electron and ion mean densities, which we will call $\tilde{n}_e$ and $\tilde{n}_i$. Thus, the picture of the plasma that we are considering in this work is that of a uniform plasma with bumps or depressions in the mean electron and ion densities around the position of the grains. Consider now that a plane electromagnetic wave $(\mathbf{E}_0,\mathbf{B}_0)$ propagates through the plasma. We assume that only the response of the electrons to the incident wave is significant, because the ions are much heavier. The incident electromagnetic field will be scattered by the plasma local heterogeneities, that is to say, by the Debye clouds, originating a scattered field $(\mathbf{E}_s,\mathbf{B}_s)$.

The Maxwell and fluid equations may be combined to give the equation for the scattered field [26]. The transverse part of the scattered field, $\mathbf{E}_T$, satisfies [a dependence of $\exp(-i\omega t)$ was assumed]

$$\nabla\times\nabla\times\mathbf{E}_T-k^2\mathbf{E}_T=i\mu_0\omega\mathbf{J}_T, \tag{2.2}$$

where $k=\omega\sqrt{\epsilon(\omega)}/c$ is the wave number entering in the plasma dispersion equation for transverse waves, $\epsilon(\omega)=1-\omega_{pe}^2/\omega^2$ being the dielectric constant of the plasma ($m_e$ is the electron mass), $\omega^2=\omega_{pe}^2+k^2c^2$, and $\mathbf{J}_T$ is the transverse part of the nonlinear current

$$\mathbf{J}=\frac{ie^2}{m\omega}\tilde{n}\mathbf{E}_0, \tag{2.3}$$

arising from the nonlinear interaction between the incident field and the Debye density perturbation. Equation (2.2) is general, whereas (2.3) is particular to a simple cold electron model. Note that $\omega_{pe}$ is defined from the equilibrium density $n_{eq,e}$, and does not take into account the effect of the density fluctuations. This is not an approximation; the fluctuation density effects are described through the nonlinear current, not by the plasma frequency.

The $\nabla\cdot\mathbf{E}$ Maxwell equation leads to the equation for the longitudinal scattered field, $\mathbf{E}_L$:

$$\mathbf{E}_L=-\frac{i}{\epsilon_0\epsilon(\omega)\omega}\mathbf{J}_L, \tag{2.4}$$

with $\mathbf{J}_L$ the longitudinal part of $\mathbf{J}$.

Equations (2.2) and (2.4) constitute the basic equations of our problem. To find the scattered transverse field we need to solve Eq. (2.2) using the technique of the Dyadic Green functions (DGFs), with the appropriate boundary conditions.

### B. The transverse scattered field in terms of the Dyadic Green function of the first kind

Following Tai [25] the DGF for (2.2) satisfies

$$\nabla\times\nabla\times\overline{\overline{G}}(\mathbf{r},\mathbf{r}')-k^2\overline{\overline{G}}(\mathbf{r},\mathbf{r}')=\overline{\overline{I}}\delta(\mathbf{r}-\mathbf{r}'). \tag{2.5}$$

The field $E_T$ can thus be obtained as

$$\begin{aligned}\mathbf{E}_T(\mathbf{r})=&\,i\omega\mu_0\int_{V'}\mathbf{J}_T(\mathbf{r}')\cdot\overline{\overline{G}}(\mathbf{r}',\mathbf{r})dV'\\&+\oint_{S'}\{[i\omega\mu_0\mathbf{H}_s(\mathbf{r}')]\cdot[\hat{\boldsymbol{n}}'\times\overline{\overline{G}}(\mathbf{r}',\mathbf{r})]\\&-[\hat{\boldsymbol{n}}'\times\mathbf{E}_T(\mathbf{r}')]\cdot\nabla'\times\overline{\overline{G}}(\mathbf{r}',\mathbf{r})\}dS',\end{aligned} \tag{2.6}$$

where $V'$ is the entire space excluding the sphere. For a conducting sphere $\hat{\boldsymbol{n}}'\times\mathbf{E}_T(\mathbf{r}')=0$ in $S'$ and if the DGF satisfies the Dirichlet boundary condition (DGF of the first kind) $\hat{\boldsymbol{n}}'\times\overline{\overline{G}}(\mathbf{r}',\mathbf{r})=0$ at the surface of the sphere, then the transverse scattered field is simply

$$\mathbf{E}_T(\mathbf{r})=i\omega\mu_0\int_{V'}\overline{\overline{G}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}_T(\mathbf{r}')dV'. \tag{2.7}$$

This result shows that for a conducting particle the scattered field does not depend on the surface charge. In the case of a dielectric or imperfectly conducting sphere the Debye field will have a contribution from the interior boundary of the volume $V'$, that is, from the surface of the dust. The field scattered by the bulk volume will induce a field on the surface of the grain, and this induced field will contribute to the total scattered field. However, if $\lambda_D \gg a$, we expect that the effect of the surface is negligible compared to the effect of the volume. Therefore, even in the case of a dielectric or imperfectly conducting sphere we still adopt (2.7) to calculate the scattered field.

The appropriate DGF for our problem is

$$\overline{\overline{G}}(\mathbf{r},\mathbf{r}') = \overline{\overline{G}}_0(\mathbf{r},\mathbf{r}') + \overline{\overline{G}}_s(\mathbf{r},\mathbf{r}'), \tag{2.8}$$

where $\overline{\overline{G}}_0(\mathbf{r},\mathbf{r}')$ is the free space DGF and $\overline{\overline{G}}_s(\mathbf{r},\mathbf{r}')$ is the scattered DGF. Thus, the free DGF generates what we will call the primary Debye field, which is the field directly radiated by the Debye cloud, and the scattered DGF generates the secondary Debye field, which is the primary field scattered by the surface of the sphere. We can describe it as a ''first Debye and then a Mie field.'' Obviously, the secondary Debye field is always smaller than both Mie and Debye primary fields and we neglect it in a first approximation.

The free space DGF is given by

$$\overline{\overline{G}}_0(\mathbf{r},\mathbf{r}') = -\frac{1}{k^2}\hat{r}\hat{r}\delta(\mathbf{r}-\mathbf{r}') + \frac{ik}{4\pi}\sum_{\sigma,m,n} C_{mn}\begin{cases}\mathbf{M}^{(1)}_{\sigma mn}(k,\mathbf{r})\mathbf{M}_{\sigma mn}(k,\mathbf{r}') + \mathbf{N}^{(1)}_{\sigma mn}(k,\mathbf{r})\mathbf{N}_{\sigma mn}(k,\mathbf{r}'), & r>r' \\ \mathbf{M}_{\sigma mn}(k,\mathbf{r})\mathbf{M}^{(1)}_{\sigma mn}(k,\mathbf{r}') + \mathbf{N}_{\sigma mn}(k,\mathbf{r})\mathbf{N}^{(1)}_{\sigma mn}(k,\mathbf{r}'), & r<r'\end{cases} \tag{2.9}$$

where the functions $\mathbf{M}_{\sigma mn}(k,\mathbf{r})$ and $\mathbf{N}_{\sigma mn}(k,\mathbf{r})$ (with $\sigma = e,o$ an index for parity and $m,n$ integers) are transverse solutions of the equation $\nabla\times\nabla\times\mathbf{F} - k^2\mathbf{F} = \mathbf{0}$. The superscript (1) is assigned to the $\mathbf{M}$ and $\mathbf{N}$ functions if they include in its expression the spherical Hankel function of the first kind, $h_n(kr)$, and no superscript is assigned if they are defined from the Bessel function of the first kind, $j_n(kr)$. Its explicit form is

$$\mathbf{M}_{\sigma mn}(k,\mathbf{r}) = \nabla\times[\psi_{\sigma mn}(k,\mathbf{r})\mathbf{r}] \tag{2.10}$$

$$\mathbf{N}_{\sigma mn}(k,\mathbf{r}) = \frac{1}{k}\nabla\times\nabla\times[\psi_{\sigma mn}(k,\mathbf{r})\mathbf{r}], \tag{2.11}$$

with $\psi_{\sigma mn}(k,\mathbf{r})$ the well known scalar solutions of $\nabla^2\psi + k^2\psi = 0$: $\psi_{\sigma mn}(k,\mathbf{r}) = z_n(kr)P_n^m(\cos\theta)f(m\phi)$, with $z_n$ a spherical function (Bessel or Hankel), $P_n^m(\cos\theta)$ an associated Legendre function and $f = \cos(\sin)$ for $\sigma = e(o)$. Finally, the coefficient of the expansion is $C_{mn} = (2-\delta_0)(2n+1)(n-m)!/n(n+1)(n+m)!$.

Besides the transverse vector solutions $\mathbf{M}$ and $\mathbf{N}$ we shall need the longitudinal solution $\mathbf{L}$:

$$\mathbf{L}_{\sigma mn}(k,\mathbf{r}) = \frac{1}{k}\nabla\psi_{\sigma mn}(k,\mathbf{r}). \tag{2.12}$$

## III. CALCULATION OF THE DEBYE FIELD

### A. Expansion of the current in the vectors M, N, and L

The use of (2.7) requires that the current is expressed in the basis constituted by the vectors $\mathbf{M}$, $\mathbf{N}$ and $\mathbf{L}$. With this expansion it is a trivial matter to perform the volume integral by the use of the orthogonal relations (A7) of Appendix A. We begin by making an explicit choice for the density perturbation and thus for the current.

For the Debye electrostatic potential around the charged sphere, $\phi$, we use the standard formula $\phi(r) = (Ze/4\pi\epsilon_0 r)\exp[-(r-a)/\lambda_D]$, where $Z$ is the number of elementary charges attached to the grain and can be either positive or negative. More realistic and complicated potentials are available in the literature (Whipple [5]) but, essentially, they are very similar to the simple Debye potential. Furthermore, the latter has great advantage for analytical work. The electron density is given by $n_e(r) = n_{eq,e} + \tilde{n} = n_{eq,e}\exp[e\phi(r)/k_bT_e] \simeq n_{eq,e}[1 + e\phi(r)/k_BT_e]$, where we have assumed that $e\phi(r)/k_BT_e \ll 1$. If the incident field is given by

$$\mathbf{E}_0 = E_0 e^{i(kz-\omega t)}\hat{\mathbf{x}}, \tag{3.1}$$

then, by use of (2.3) we obtain for the current ($e^{-i\omega t}$ omitted)

$$\mathbf{J}(\mathbf{r}) = J\frac{e^{-r/\lambda_D}}{r}e^{ikz}\theta(r-a)\hat{\mathbf{x}}, \quad J = i\frac{e^2E_0}{4\pi m}\frac{Ze^{a/\lambda_D}}{\omega\lambda_{De}^2}, \tag{3.2}$$

where $\theta(r-a)$ is the unit step function centered at $r=a$ ($=1$ if $r>a$ and $=0$ if $r\leqslant a$), introduced to take into account the obvious fact that the electron plasma current only exists outside the sphere. Note that $J$ depends on the plasma and the incident wave parameters.

Now we make an eigenfunction expansion of the current:

$$\begin{aligned}\mathbf{J}(\mathbf{r}) &= J\frac{e^{-r/\lambda_D}}{r}e^{ikz}\theta(r-a)\hat{\mathbf{x}} \\ &= \int_0^\infty dh\sum_{\sigma,m,n}[a_{\sigma mn}(h)\mathbf{M}_{\sigma mn}(h,\mathbf{r}) \\ &\quad + b_{\sigma mn}(h)\mathbf{N}_{\sigma mn}(h,\mathbf{r}) + c_{\sigma mn}(h)\mathbf{L}_{\sigma mn}(h,\mathbf{r})].\end{aligned} \tag{3.3}$$

To determine the coefficients of the decomposition we multiply successively both sides of (3.3) by $\mathbf{M}$, $\mathbf{N}$, and $\mathbf{L}_{\sigma'm'n'}(h',\mathbf{r})$, integrate in $\mathbf{r}$ and use the orthogonality relations (A7) of the Appendix. For the $b$ coefficients, for example, we obtain

$$b_{\sigma mn}(h)=J\frac{h^2C_{mn}}{2\pi^2}\hat{\mathbf{x}}\cdot\int_a^\infty dr\, re^{-r/\lambda_{De}}\int d\Omega e^{ikz}\mathbf{N}_{\sigma mn}(h,\mathbf{r}),\tag{3.4}$$

where $\mathbf{r}=(r,\theta,\phi)$ and $d\Omega=\sin\theta d\theta d\phi$; we are using the standard spherical coordinates, $\theta$ being the angle between $\mathbf{r}$ and the $z$ axis (parallel to $\mathbf{k}$). To perform the angular integral we decompose $\mathbf{N}_{\sigma mn}(h,\mathbf{r})$ in the vector spherical harmonics [24], $\mathbf{P}^{\sigma}_{mn}(\theta,\phi)$, $\mathbf{B}^{\sigma}_{mn}(\theta,\phi)$, and $\mathbf{C}^{\sigma}_{mn}(\theta,\phi)$, according to (A3) of the Appendix. Then, we use the fact that

$$\int d\Omega e^{ikz}\mathbf{P}^{\sigma}_{mn}(\theta,\phi)=4\pi i^{n-1}\mathbf{L}_{\sigma mn}(k,u=0,v)$$

and

$$\int d\Omega e^{ikz}\mathbf{B}^{\sigma}_{mn}(\theta,\phi)=4\pi i^{n-1}\mathbf{N}_{\sigma mn}(k,u=0,v)/\sqrt{n(n+1)},$$

where $(k,u=0,v)$ are the angular coordinates of the vector $\mathbf{k}=k\hat{\mathbf{z}}$. Splitting again $\mathbf{N}_{\sigma mn}(k,u=0,v)$ and $\mathbf{L}_{\sigma mn}(k,u=0,v)$ in its vector harmonic components, using $\mathbf{P}^{\sigma}_{mn}(u=0,v)=\hat{\mathbf{z}}\delta_{\sigma e}\delta_{m0}$, $\mathbf{B}^{e}_{mn}(u=0,v)=-(1/2)\sqrt{n(n+1)}\hat{\mathbf{x}}\delta_{m1}$, $\mathbf{B}^{o}_{mn}(u=0,v)=-(1/2)\sqrt{n(n+1)}\hat{\mathbf{y}}\delta_{m1}$ and the recurrence relations for the spherical Bessel functions, finally one obtains the expression for the $b$ coefficients:

$$b_{\sigma mn}(h)=\delta_{\sigma e}\delta_{m1}J\frac{i^{n+1}}{\pi}\frac{2(2n+1)}{n(n+1)}h^2I_2^n(h,k),\tag{3.5}$$

with

$$I_2^n(h,k)=\frac{1}{2n+1}\int_a^\infty dr re^{-r/\lambda_D}[(n+1)j_{n-1}(hr)j_{n-1}(kr)+nj_{n+1}(hr)j_{n+1}(kr)].\tag{3.6}$$

Following similar steps one obtains, for the $a$ and $c$ coefficients,

$$a_{\sigma mn}(h)=\delta_{\sigma o}\delta_{m1}J\frac{i^{n+2}}{\pi}\frac{2(2n+1)}{n(n+1)}h^2I_1^n(h,k),\tag{3.7}$$

$$c_{\sigma mn}(h)=\delta_{\sigma e}\delta_{m1}J\frac{i^{n+1}}{\pi}2h^2I_3^n(h,k),\tag{3.8}$$

with

$$I_1^n(h,k)=\int_a^\infty dr re^{-r/\lambda_D}j_n(hr)j_n(kr),\tag{3.9}$$

$$I_3^n(h,k)=\int_a^\infty dr re^{-r/\lambda_D}[j_{n-1}(hr)j_{n-1}(kr)-j_{n+1}(hr)j_{n+1}(kr)].\tag{3.10}$$

### B. The transverse Debye field

Now that we have the current term decomposed in the vectors $\mathbf{M}$, $\mathbf{N}$ and $\mathbf{L}$, as stated by Eq. (3.3), we can go back to (2.7) and perform the integral $\int_{V'}\overline{\overline{G}}_0(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}_T(\mathbf{r}')dV'$ using the orthogonality relations for these vectors. The integration can be performed over the entire space because the current vanishes for $r<a$. The orthogonality relations produce the factor $\delta(h-k)$ to be used under the $h$ integration in (3.3). Because we are interested in the field far away from the sphere, we can assume that the point of observation, located at $\mathbf{r}$, satisfies $r>r'$, where $\mathbf{r}'$ is a source point. Therefore we take the upper branch of (2.9) and neglect the $\delta$ function of this expression. The final result is (we add a superscript $D$, for Debye)

$$\mathbf{E}_T^D(\mathbf{r})=J\frac{\mu_0\sqrt{\epsilon(\omega)}}{c}\omega^2\sum_n\frac{(2n+1)i^n}{n(n+1)}\times[I_1^n(k)\mathbf{M}^{(1)}_{o1n}(k,\mathbf{r})-iI_2^n(k)\mathbf{N}^{(1)}_{e1n}(k,\mathbf{r})].\tag{3.11}$$

The asymptotic form $(kr\to\infty)$ is obtained by using $h_n(z)\to i^{-(n+1)}e^{iz}/z$ $(z\to\infty)$,

$$\mathbf{E}_T^D(\mathbf{r})\overset{kr\to\infty}{\sim}E_0r_0\frac{Ze^{a/\lambda_D}}{\lambda_{De}^2}\frac{e^{ikr}}{r}\sum_n\frac{2n+1}{\sqrt{n(n+1)}}\times[I_2^n(k)\mathbf{B}^e_{1n}(\theta,\phi)+I_1^n(k)\mathbf{C}^o_{1n}(\theta,\phi)],\tag{3.12}$$

where $r_0=e^2/(4\pi\epsilon_0mc^2)$ is the classical electron radius and we have defined $I_i^n(k)\equiv I_i^n(k,k)$, for $i=1,2,3$. This is the central result of this work, to be compared with the asymptotic form for the Mie field (entirely transverse):

$$\mathbf{E}^M(\mathbf{r})\overset{kr\to\infty}{\sim}iE_0\frac{e^{ikr}}{kr}\sum_n\frac{2n+1}{\sqrt{n(n+1)}}\times[a_n^M\mathbf{B}^e_{1n}(\theta,\phi)+b_n^M\mathbf{C}^o_{1n}(\theta,\phi)].\tag{3.13}$$

The Mie coefficients are very well known from the litterature. For a dielectric sphere $a_n^M$ and $b_n^M$ are given by [13]

$$a_n^M=\frac{m\psi_n(mx)\psi_n'(x)-\psi_n(x)\psi_n'(mx)}{m\psi_n(mx)\xi_n'(x)-\xi_n(x)\xi_n'(mx)}$$

and

$$b_n^M=\frac{\psi_n(mx)\psi_n'(x)-m\psi_n(x)\psi_n'(mx)}{\psi_n(mx)\xi_n'(x)-m\xi_n(x)\xi_n'(mx)},$$

where $\psi_n(t)=tj_n(t)$, $\xi_n(t)=th_n(t)$, $m=n_1/n$, $n_1$ being the (complex) refractive index of the particle and $n$ that of the medium, and $x=2\pi na/\lambda$.

Thus, besides the longitudinal field (discussed below), the Debye field and the Mie field have the same form, the quantitative differences lying on the coefficients of the expansion.

### C. The longitudinal Debye field

Contrary to the Mie field, the Debye field has a longitudinal component while propagating in the plasma. From (2.4), (3.3), and (3.8) one obtains

$$\mathbf{E}_L^D(\mathbf{r}) = \frac{2J}{\pi\epsilon_0\epsilon(\omega)\omega}\sum_n i^n\int_0^\infty dh h^2 I_3^n(h,k)\mathbf{L}_{e1n}(h,\mathbf{r}). \tag{3.14}$$

The calculations here proceed differently from the transverse case. We cannot use directly the asymptotic expression for the Bessel function $j_n(hr)$ included in $\mathbf{L}_{e1n}(h,\mathbf{r})$, because it is under integration. Instead, we can split $\mathbf{L}_{e1n}$ in a product of Bessel function ($j_n$) and vector spherical harmonics ($\mathbf{P}_{mn}^\sigma$ and $\mathbf{B}_{mn}^\sigma$, see Appendix A) and change the order of integration between the $h$ integral and the integral of $I_3^n$, ending up with an expression involving integrals of the type $\int_0^\infty dh h j_{n-1}(h\rho)j_n(hr) = (\pi/2)\rho^{n-1}/r^{n+1}$ [if $0<\rho<r$], $=\pi/4r^2$ [if $0<\rho=r$] and $=0$ [if $\rho>r>0$] ($\rho$ is the variable of integration of $I_3^n$). The field $\mathbf{E}_L^D$ is then obtained for arbitrary $r$:

$$\begin{aligned}\mathbf{E}_L^D(\mathbf{r}) = iE_0r_0\frac{Ze^{a/\lambda_D}}{k^2\lambda_{De}^2}\sum_n i^n\Bigg\{&[\sqrt{n(n+1)}\mathbf{B}_{1n}^e(\theta,\phi)\\ &-(n+1)\mathbf{P}_{1n}^e(\theta,\phi)]\int_a^r d\rho e^{-\rho/\lambda_D}j_{n-1}(k\rho)\frac{\rho^n}{r^{n+2}}\\ &-[\sqrt{n(n+1)}\mathbf{B}_{1n}^e(\theta,\phi)+n\mathbf{P}_{1n}^e(\theta,\phi)]\\ &\times\int_r^\infty d\rho e^{-\rho/\lambda_D}j_{n+1}(k\rho)\frac{r^{n-1}}{\rho^{n+1}}\Bigg\}.\end{aligned} \tag{3.15}$$

In the limit $r\to\infty$ the first integral becomes dominant. Approximating it by $\int_0^\infty$ one obtains

$$\begin{aligned}\mathbf{E}_L^M(\mathbf{r}) \sim E_0Ze^{a/\lambda_D}\frac{kr_0}{k^2\lambda_{De}^2}&\sum_{n=1}^\infty 2^{n-1}(n-1)!\frac{i^{n+1}}{(kr)^{n+2}}\\ &\times\frac{(k\lambda_D)^n}{(1+k^2\lambda_D^2)^n}[\sqrt{n(n+1)}\mathbf{B}_{1n}^e(\theta,\phi)\\ &-(n+1)\mathbf{P}_{1n}^e(\theta,\phi)].\end{aligned} \tag{3.16}$$

Due to the strong dependence on $1/r^{n+2}$ the first term ($n=1$) is dominant. Thus, the asymptotic longitudinal field can be approximated by the first term of the series, which can be written as

$$\begin{aligned}\mathbf{E}_L^D(\mathbf{r}) \sim E_0\frac{Z}{1+\delta}e^{a/\lambda_D}\frac{kr_0}{(kr)^3}\frac{2}{1+k^2\lambda_D^2}\\ \times\left(\hat{\mathbf{r}}\sin\theta\cos\phi-\frac{1}{2}\hat{\boldsymbol{\theta}}\cos\phi\cos\theta+\frac{1}{2}\hat{\boldsymbol{\phi}}\sin\phi\right),\end{aligned} \tag{3.17}$$

with

$$\delta = \frac{T_e n_{eq,i}}{T_i n_{eq,e}} \tag{3.18}$$

an important parameter to be discussed in Sec. IV. Several comments are now in order. This field is proportional to $1/r^3$ and thus it is negligible in comparison to the asymptotic transverse field. As can be seen from expression (3.17), the longitudinal field is not strictly longitudinal, in the sense that it is not all parallel to $\hat{r}$. This is a simple consequence of the fact that the vector harmonic $\mathbf{L}$ also is not parallel to $\hat{r}$. However, the $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\phi}}$ components depend on $1/(kr)^3$ and do not imply any correction to the expression for the transverse field. From (3.17) it is clear that the longitudinal field is not propagating as an outgoing wave but rather as a kind of evanescent field.

## IV. PHYSICAL PROPERTIES OF THE DEBYE-MIE FIELD

### A. Simplified expressions and limiting cases

With generality the $I_1^n(k)$ coefficient can be written as

$$I_1^n(k) = \frac{1}{2k^2}Q_n\left(1+\frac{1}{2k^2\lambda_D^2}\right)-\int_0^a dr r e^{-r/\lambda_D}j_n^2(kr), \tag{4.1}$$

where we have used the result $\int_0^\infty dx x e^{-ax}j_n(bx)j_n(cx) = (1/2bc)Q_n[(a^2+b^2+c^2)/(2bc)]$ and $Q_n(x)$ is the Legendre polynomial of the second kind. It can be easily seen that if $a\ll\lambda_D,\lambda$ (where $\lambda$ is the wavelength of the electromagnetic field in the plasma), and for any relation between $\lambda_D$ and $\lambda$ one can neglect the second term of the right-hand side of (4.1) comparatively to the first one. This is the most common case, and it is equivalent to considering the dust as a point particle. The scattered Debye field (3.11) is then simplified to

$$\begin{aligned}\mathbf{E}_T^D(\mathbf{r}) = \frac{1}{2}E_0r_0Z\frac{1}{(k\lambda_{De})^2}\frac{e^{ikr}}{r}\sum_n\frac{2n+1}{\sqrt{n(n+1)}}\\ \times\left[Q_n\left(1+\frac{1}{2(k\lambda_D)^2}\right)\mathbf{C}_{1n}^o(\theta,\phi)\right.\\ \left.+R_n\left(1+\frac{1}{2(k\lambda_D)^2}\right)\mathbf{B}_{1n}^e(\theta,\phi)\right],\end{aligned} \tag{4.2}$$

with $R_n(x) = [(n+1)Q_{n-1}(x)+nQ_{n+1}(x)]/(2n+1)$.

In the Rayleigh limit we will consider $\lambda\gg\lambda_D,a$ and any relation between $a$ and $\lambda_D$, that is, it is indifferent to have $\lambda_D>a$ or $<a$. In this limit we can use in (4.1) the asymptotic expressions for large arguments of $Q_n$ and small arguments of $j_n$. It turns out that the dominant term in the series (3.12) is that of $\mathbf{B}_{11}^e$. After some manipulations with the complete and incomplete $\gamma$ functions one obtains

$$\mathbf{E}_T^D(\mathbf{r}) \xrightarrow{k\lambda_{De}\ll1} E_0r_0Z\frac{1+\epsilon}{1+\delta}\frac{e^{ikr}}{r}(\cos\phi\cos\theta\hat{\boldsymbol{\theta}}-\sin\phi\hat{\boldsymbol{\phi}}), \tag{4.3}$$

where $\epsilon = a/\lambda_D$ and $\delta$ was already introduced in (3.18). The factor $\epsilon$ appears in the expression because no magnitude relation was assumed between $a$ and $\lambda_D$. If $a = \lambda_D$ it represents a considerable enhancement (a factor of 4 in the cross section) relatively to the Rayleigh pointlike case, in which $\lambda_D \gg a \simeq 0$. Physically it just means that if $a$ increases with $\lambda_D$ constant, then the Debye cloud expands, having a higher cross section. The factor $\delta$ is due to the factor $\lambda_D^2/\lambda_{De}^2$, which appears because the electrons have a distribution according to the potential $\propto \exp(-r/\lambda_D)$, but their dynamics leads to $\lambda_{De}$. In other words, the dust is screened by electrons and ions, but only the electrons do the scattering. If $T_e = T_i$ and $Z < 0$, for example, the charge of the grain is screened by $Z/2$ ions and (by the absence of) $Z/2$ electrons. In general, the number of electrons missing in the Debye sphere is $Z/(1+\delta)$. In the rest of this work we will refer to $Z/(1+\delta)$ as the number of shielding electrons in the Debye sphere, having in mind that this number can be negative for a negative dust. If $T_e \gg T_i$ and the grain density is low (so that $n_{eq,i} \simeq n_{eq,e}$) $\delta$ represents a severe suppression of the Debye field. As expected, the amplitude of the scattered wave does not depend on $k$, which is consistent with the fact that the incident wave does not ''see'' the inner structure of the Debye cloud; the $Z/(1+\delta)$ electrons on it scatter coherently. The DS cross section becomes constant, contrary to the Mie case, where the long wavelength regime leads to a $1/\lambda^4$ behavior. However, the angular dependence is exactly the same in both cases.

Finally, concerning the small wavelength limit, no simple expression can be derived. The discussion is simpler if we consider the point particle case (4.2). If $k\lambda_D \to \infty$, then the argument of the Legendre functions tends to 1, where it has a logarithmic singularity. Thus, an increasing number of terms is needed to compute the Debye field as $k\lambda_D \to \infty$, and no simple closed form can be derived. However, the prefactor $1/(k\lambda_{De})^2$ goes to zero faster than the Legendre functions tend to infinity. Thus, we expect that the small wavelength limit the Debye field has the form

$$\mathbf{E}_T^D(\mathbf{r}) = \frac{1}{2} E_0 r_0 Z \frac{1}{(k\lambda_{De})^2} \frac{e^{ikr}}{r} \mathbf{F}(k\lambda_D, \theta, \phi), \tag{4.4}$$

where $\mathbf{F}(k\lambda_D, \theta, \phi)$ is a (vector) function with a weak dependence on $k\lambda_D$. This form of the Debye field is verified numerically, the plot of the Debye cross section being well fitted by a $1/\lambda^4$ curve in the small wavelength regime. By opposition with the Mie process, where the scattering cross section tends to a constant value $\pi a^2$ in the small wavelength limit, the Debye field vanishes very fast because the electrons in the Debye cloud are scattering the incident radiation out of phase relative to each other.

### B. Total cross section

The total field $\mathbf{E}^{\text{tot}} = \mathbf{E}^D + \mathbf{E}^M$ is the sum of the Debye and Mie fields. It can be written in the form of (3.13), with the substitution $a_n^M \to a_n^{\text{tot}} = a_n^M + a_n^D$, $b_n^M \to b_n^{\text{tot}} = b_n^M + b_n^D$, with $a_n^D, b_n^D = -ikr_0 Z \exp(a/\lambda_D) I_{2,1}^n(k)/\lambda_{De}^2$. Therefore, the formal results of the Mie theory concerning cross sections can be used almost directly, with minor changes. This is the reason why in the following we present only the results accompanied by the minimum additional information.

Because the total magnetic field satisfies $\mathbf{H}^{\text{tot}} \to (k/\mu_0\omega)\hat{\boldsymbol{r}} \times \mathbf{E}^{\text{tot}}$ $(kr \to \infty)$, the time averaged Poynting vector in the asymptotic region is $\langle \mathbf{S} \rangle = \text{Re}(\mathbf{E}^{\text{tot}} \times \mathbf{H}^{\text{tot}\,\star})/2 = \sqrt{\epsilon_0 \epsilon(\omega)/\mu_0}\,\hat{\boldsymbol{r}} |\mathbf{E}^{\text{tot}}|^2/2$. The irradiance is simply $I = \langle |\mathbf{S}| \rangle$ and we define $I_\parallel$ ($I_\perp$) as the irradiance for the case when the incident field is parallel (perpendicular) to the scattering plane (defined by the incident and scattered wave vectors). Because we have chosen the incident field to be polarized in the X direction, $I_\parallel$ is computed from $\mathbf{E}^{\text{tot}}(\text{X pol}, \theta, \phi = 0)$ and $I_\perp$ is computed from $\mathbf{E}^{\text{tot}}(\text{Y pol}, \theta, \phi = 0) = \mathbf{E}^{\text{tot}}(\text{X pol}, \theta, \phi = -\pi/2)$.

The total cross section is calculated from the total scattered power $P$, $\sigma^{\text{tot}} = P/I_0 = \int_A \langle \mathbf{S} \rangle \cdot d\mathbf{A}/I_0$, where $I_0$ is the incident irradiance. If $A$ is a spherical surface in the asymptotic region, $\sigma = \int_A I dA/I_0$, which, by use of the orthogonal relations between the $\mathbf{B}$ and $\mathbf{C}$ vectors gives

$$\sigma^{\text{tot}} = \sigma^M + \sigma^D + \sigma^{MD}, \tag{4.5}$$

with

$$\sigma^{M,D} = \frac{2\pi}{k^2} \sum_{n=1}^{\infty} (2n+1)(|a_n^{M,D}|^2 + |b_n^{M,D}|^2) \tag{4.6}$$

and $\sigma^{MD}$ an interference cross section given by

$$\sigma^{MD} = -4\pi Z e^{a/\lambda_D} \frac{kr_0}{(k\lambda_{De})^2} \times \text{Im}\left\{ \sum_{n=1}^{\infty} (2n+1)[I_2^n(k) a_n^M + I_1^n(k) b_n^M] \right\}. \tag{4.7}$$

Recalling that $Z$ can be positive or negative according to the dust charge sign, we see that the interference term introduces a separation between the total cross sections for oppositely charged dust grains. Therefore, even if the model for the potential around the grain leads to the same form of the perturbation density, as is the present case, the interference term ''raises the degeneracy,'' introducing a difference between the total cross sections. Obviously, this is due to the fact that the sign of the perturbation density changes if the grain charge changes, leading to a phase difference of $\pi$ in the Debye field. This interference term can only be significant if the Debye and Mie fields are of the same order of magnitude. Typically, this happens if $a \ll \lambda$, that is, in the Rayleigh-Mie regime, where the dominant coefficient is $a_1^M$, whose imaginary part is negative. Thus, we conclude that in the cases of interest the interference is constructive for a positive grain and destructive for a negative one.

In the Rayleigh limit for the Debye process ($\lambda \gg \lambda_D, a$) one obtains the Debye cross section directly from (4.3):

$$\sigma^D(k\lambda_D \ll 1) = Z^2 \sigma_0 \left( \frac{1+\epsilon}{1+\delta} \right)^2, \tag{4.8}$$

where $\sigma_0 = 8\pi r_0^2/3$ is the Thomsom cross section. This is in agreement with the fact that the $Z$ electrons in the Debye cloud are scattering coherently. However, as previously remarked, the correction factor $[(1+\epsilon)/(1+\delta)]^2$ must be taken into account because it can represent a strong suppression of the coherent effect. Note that $\delta$ is at least 1, so that if $\epsilon$ is negligible (as in most cases), it introduces at least a suppression factor of 4.

### C. Differential cross section. A possible way to determine the factors $Z/(1+\delta)$ and $\lambda_D$

The angular variation of the scattered field can be adequately described by the radiant intensity $dP/d\Omega$ (units $\mathrm{W\,sr^{-1}}$), defined by $P=\int_{4\pi}(dP/d\Omega)d\Omega \Rightarrow dP/d\Omega=R^2I$, where $R$ is the radius of the spherical surface $A$ considered above, or equivalently by the differential cross section (dimensions $\mathrm{m^2\,sr^{-1}}$), defined from $\sigma=\int_{4\pi}(d\sigma/d\Omega)d\Omega \Rightarrow d\sigma/d\Omega=R^2I/I_0$. Thus, the differential cross section can be also be split into the sum of its parallel and perpendicular components, which are

$$\left(\frac{\partial\sigma}{\partial\Omega}\right)_{\parallel}=\frac{1}{k^2}\left|\sum_n\,[a_n^{\mathrm{tot}}p_n(\theta)+b_n^{\mathrm{tot}}q_n(\theta)]\right|^2, \tag{4.9}$$

$$\left(\frac{\partial\sigma}{\partial\Omega}\right)_{\perp}=\frac{1}{k^2}\left|\sum_n\,[a_n^{\mathrm{tot}}q_n(\theta)+b_n^{\mathrm{tot}}n_n(\theta)]\right|^2, \tag{4.10}$$

with $p_n(\theta)=[nP_{n+1}^1(\cos\theta)/(n+1)-(n+1)P_{n-1}^1(\cos\theta)/n]/\sin\theta$ and $q_n(\theta)=(2n+1)P_n^1(\cos\theta)/n(n+1)\sin\theta$. Plots of these quantities are shown in Sec. IV E

For most of the experimental situations the measure of the total cross section is not possible, but only the differential cross section in some allowed/accessed directions. Naturally, the cases $\theta=0$ (forward scattering) and $\theta=\pi$ (backscattering) are of particular interest, not only for experimental purposes, but also because the infinite series of the Debye field can be summed exactly, leading to a simple expression, independent of the relation between $\lambda$ and $\lambda_D$.

For $\theta=0$ the associated Legendre functions satisfy $\lim_{\theta\to 0}P_n^1(\cos\theta)/\sin\theta=-n(n+1)/2$, which implies that the parallel and perpedicular differential cross sections are equal and

$$\frac{d\sigma}{d\Omega}(\theta=0)=\frac{1}{4k^2}\left|\sum_{n=1}^{\infty}\,(2n+1)(a_n^{\mathrm{tot}}+b_n^{\mathrm{tot}})\right|^2. \tag{4.11}$$

We are interested in describing the wavelength range where the Debye field is of the same order of magnitude or greater than the Mie field. This happens when the Mie field is already in the Rayleigh regime, and so we retain only the dominant Mie term, which is $a_1^M$. Thus, the sum to perform is now restricted to Debye terms. Using the result $\Sigma_{n=0}^{\infty}(2n+1)Q_n(1+\epsilon)=1/\epsilon$ (which can be derived from the well known general expression $\Sigma_{n=0}^{\infty}(2n+1)Q_n(t)P_n(z)=(t-z)^{-1}$, valid for complex $z$ inside an ellipse passing through $t$, and having foci at the points $\pm 1$ [24]) and the explicit form of the Mie coefficient $a_1^M$, given above, one obtains finally

$$\frac{d\sigma}{d\Omega}(\theta=0)=\left|\frac{m^2-1}{m^2+2}\right|^2 a^6k^4+r_0^2\left(\frac{Z}{1+\delta}\right)^2 +2\,\mathrm{Re}\left(\frac{m^2-1}{m^2+2}\right)r_0\frac{Z}{1+\delta}a^3k^2. \tag{4.12}$$

It is important to note that if the particle properties ($a$ and $m$) are known, then $d\sigma/d\Omega(\theta=0)$ depends only on the parameter $Z/(1+\delta)$, the number of electrons in the Debye sphere.

For $\theta=\pi$ a similar deduction can be made using $\lim_{\theta\to\pi}P_n^1(\cos\theta)/\sin\theta=(-)^n n(n+1)/2$ and $\Sigma_{n=0}^{\infty}(2n+1)(-)^nQ_n(1+\epsilon)=1/(2+\epsilon)$, with the result

$$\frac{d\sigma}{d\Omega}(\theta=\pi)=\left|\frac{m^2-1}{m^2+2}\right|^2 a^6k^4++r_0^2\left(\frac{Z}{1+\delta}\right)^2\frac{1}{(1+4k^2\lambda_D^2)^2} +2\,\mathrm{Re}\left(\frac{m^2-1}{m^2+2}\right)r_0\frac{Z}{1+\delta}\,\frac{a^3k^2}{1+4k^2\lambda_D^2}. \tag{4.13}$$

Therefore, in the backscattering case there is an extra dependency on the parameter $1+4k^2\lambda_D^2$ relative to the forward scattering case and the information on the parameter $Z/(1+\delta)$ is mixed with the information on the plasma parameter $\lambda_D$.

These results suggest a way to determine the parameters $Z/(1+\delta)$ and $\lambda_D$. The curves $d\sigma/d\Omega_{\theta=0,\pi}(k)$ have a minimum. Taking the derivative of (4.12) and (4.13) in order to $k$ and equating the result to zero one obtains two expressions for $Z/(1+\delta)$. The forward case gives

$$\frac{Z}{1+\delta}=-\frac{|\mu|^2a^3k_{0,min}^2}{\mathrm{Re}(\mu)r_0}, \tag{4.14}$$

where $k_{0,min}$ is the value of $k$ where the differential cross section for $\theta=0$ attains the minimum and we have defined $\mu=(m^2-1)/(m^2+2)$. The backscattering case gives

$$\frac{Z}{1+\delta}=\frac{\mathrm{Re}(\mu)a^3(1+4k_{\pi,min}^2\lambda_D^2)}{8r_0\lambda_D^2}\left[1\pm\sqrt{1+16\frac{|\mu|^2}{[\mathrm{Re}(\mu)]^2}k_{\pi,min}^2\lambda_D^2(1+4k_{\pi,min}^2\lambda_D^2)}\right], \tag{4.15}$$

where $k_{\pi,min}$ is the value of $k$ where the differential cross section for $\theta=\pi$ attains its minimum. The forward scattering case allows the determination of $Z/(1+\delta)$ if one knows the properties of the spheres alone, whereas the backscattering case needs the previous knowledge of the Debye length. However, if both possibilities are experimentally available, we can equate the right-hand sides of (4.14) and (4.15), and it turns out that $\lambda_D$ is the solution of the polynomial equation

$$1+4\left(\frac{|\mu|^2}{[\mathrm{Re}(\mu)]^2}k_{0,min}^2+k_{\pi,min}^2\right)\lambda_D^2 - \frac{k_{\pi,min}^2}{k_{0,min}^2}(1+4k_{\pi,min}^2\lambda_D^2)^3=0. \quad (4.16)$$

As discussed in the introduction and in Sec. VI, this method applies to a collection of grains if, first, $n_d^{-1/3}>\lambda_D$ and, second, the grains are monodisperse in size.

### D. Degree of polarization

Finally, we define the degree of polarization of the scattered field for a totally depolarized incident field. As usual, it is given by $P=(I_\perp-I_\|)/(I_\perp+I_\|)$.

The curve $P(\theta)$ is the same for all incident wavelengths in the Rayleigh regime for the MS. This is due to the fact that our treatment is single scattering, which means that the scattered field by the Debye cloud preserves the polarization curve $P(\theta)$ of an individual electron. It is easy to see that if two electrons, 1 and 2, have individual degrees of polarization $p=(I_{\perp i}-I_{\|i})/(I_{\perp i}+I_{\|i})$, $i=1,2$ (at a given angle $\theta$), then $I_{\perp T}\propto(\mathbf{E}_{\perp 1}+\mathbf{E}_{\perp 2})^2$ and $I_{\|T}\propto(\mathbf{E}_{\|1}+\mathbf{E}_{\|2})^2$ still satisfy $p=(I_{\perp T}-I_{\|T})/(I_{\perp T}+I_{\|T})$ for the same angle. The reasoning can be extended to an arbitrary number of electrons. The polarization curve for the Raleigh-Mie field is the same as that for an electron and so the previous argument also applies to a sphere and cloud of electrons. As a final conclusion, the polarization curve in the Born approximation is expected to be always the same for all incident wavelengths, and equal to $P(\theta)=(1-\cos^2\theta)/(1+\cos^2\theta)$. This is confirmed numerically.

### E. Some numerical examples

In all of the plots discussed below it is assumed that $\delta=1$ and the complex refractive index of the particle is $n=2+i$.

#### *1. Cross sections*

In Fig. 1 we present a plot of the total cross section for a charged spherical particulate immersed in a typical plasma processing chamber environment [12]. The radius of the grain is $a=10$ nm, the Debye length was assumed to be $\lambda_D=1$ $\mu$m and the charge of the grain is $|Z|=100$. The curves represent the variation with the incident wavelength $\lambda$ of the quantities $\sigma^{\mathrm{tot}}$ for a positive and for a negative charge, $\sigma^M$, $\sigma^D$ and $\sigma^{MD}$. These calculations follow from expressions (4.5), (4.6) and (4.7), with the DS coefficients calculated neglecting the integral $\int_0^a$ in (4.1).

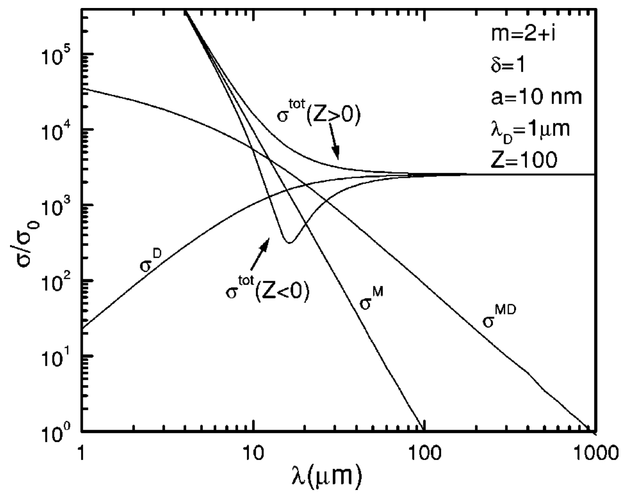


FIG. 1. Cross sections for Mie and Debye scattering (normalized to the Thomson cross section) in a typical environment of a plasma processing chamber ($\lambda_D=1$ $\mu$m, $a=10$ nm, and $|Z|=100$) as a function of the incident wavelength. The interference term $\sigma_{MD}$ is also represented and leads to a difference between positively and negatively charged grains in an intermediate regime of $\lambda\sim10$ $\mu$m. Due to this interference, $\sigma^D$ is almost one order of magnitude smaller than $\sigma_0 Z^2$. The Rayleigh limit for Debye scattering is constant.

It is clear that in the wavelength range 8–40 $\mu$m the interference cross section $\sigma^{MD}$ introduces a shift between the total cross section curves for the positive and negative dusts. In the case of a negative dust, this effect almost spoils completely the scattering enhancement effect in the 10-$\mu$m range. Furthermore, in this case a minimum is observed, whereas in the positive charge case the curve of $\sigma^{\mathrm{tot}}$ is monotonic. This happens precisely in the transition region where Mie and Debye processes are equally important. For $\lambda<8$ $\mu$m the MS dominates whereas for $\lambda>40$ $\mu$m it is the Debye process that dominates.

Thinking of an environment closer to that of a dusty crystal experiment [10], the value of $a$ should be taken as approximately 1 $\mu$m, the Debye length as $\sim100$ $\mu$m, and the charge maybe as high as $10^3$ or even $10^4$. The transition region would be shifted towards the millimeter wavelength range because $\sigma^M\propto a^6\lambda^{-4}$, but the Rayleigh limit for the Debye cross section would rise to $\sim10^6,10^8\sigma_0$.

In some conditions the interference term is negligible. This can be seen in Fig. 2, where we assumed a dust in a typical space plasma: $a=1$ $\mu$m, $\lambda_D=1$ m, and $|Z|=1000$. In this case the transition region is given by $\lambda\sim0.1$ m, and the MS cross section is entirely given by the first (Rayleigh) term. On the contrary, for the DS we are still in the region $\lambda\ll\lambda_D$ and a large number of small terms is required to compute the cross section. Therefore, because $\sigma^{MD}$ is thesum of the cross terms of the type $a_n^M b_n^D$ and $b_n^M a_n^D$, only the first term will contribute (because $a_1^M\gg b_1^M, a_n^M, b_n^M$, $i\geqslant2$), and not much, because $a_1^D$ and $b_1^D$ are small and of the same order of magnitude of the next $a_n^D$, $b_n^D$ hundred terms. Therefore, in this case the splitting introduced by $\sigma^{MD}$ is negligible and the total cross section for positive and negative grains is identical. Finally, note that the Debye cross section falls to zero as $\lambda\to0$ slightly slower than the Mie

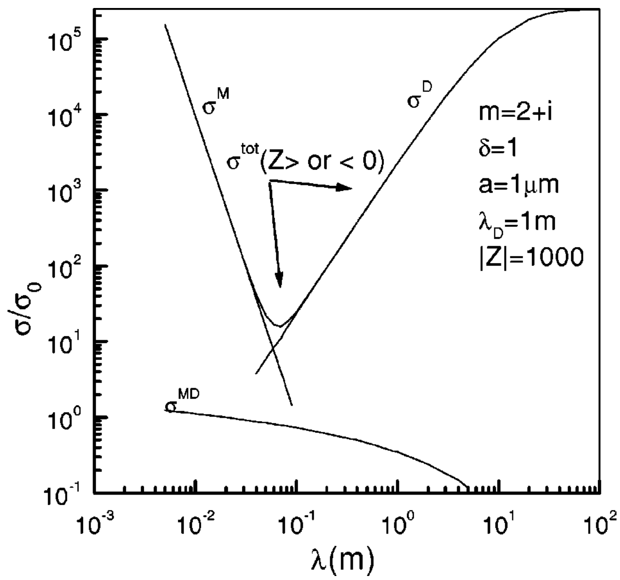


FIG. 2. Cross sections (normalized to the Thomson cross section) for Mie and Debye scattering in a typical space plasma ($\lambda_D = 1$ m, $a=1$ $\mu$m, and $|Z|=1000$), as a function of the wavelength. The interference term $\sigma^{MD}$ is not important in this case.

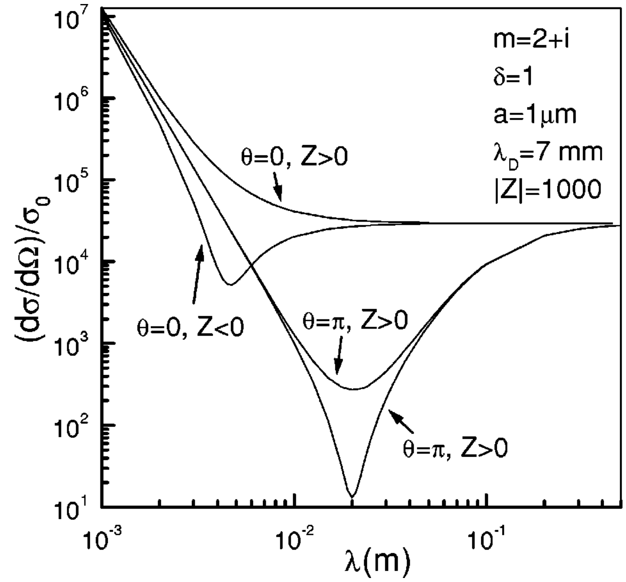


FIG. 4. Differential cross sections for $\theta=0$ (forward scattering) and $\theta=\pi$ (backscattering), for $Z>0$ and $Z<0$. The values of the parameters used here correspond roughly to the ionosphere: $\lambda_D = 7$ mm, $a=1$ $\mu$m, and $|Z|=1000$. The minimums do not coincide. Again, the difference between $d\sigma/d\Omega$ for positive and negative grains is very large in the transition region.

cross section does as $\lambda\rightarrow\infty$. We know that the latter follows a $1/\lambda^4$ dependence, which means that $\sigma^D$ is falling as $\lambda^{4-\alpha}$, with $\alpha$ a small number, in accordance with (4.4).

### *2. Differential cross sections*

In Fig. 3 we show the angular dependence of the scattering. The plots show $I_{\parallel}$ and $I_{\perp}$ normalized by its maximum values (at $\theta=0$) for $a=1\,\mu$m, $\lambda_D=1$m (typical space plasma), $Z=-1000$ and for three different values of the incident wavelength: $\lambda=1$, 10 and 100 m. The calculations follow from (4.9) and (4.10).

In the long wavelength limit the plots reproduce the shape of the Raleigh-Mie curves, that is, inverse bell shaped for the parallel irradiance and flat for the perpendicular irradiance. When the incident wavelength decreases the curves show a strong tendency to peak in the forward direction.

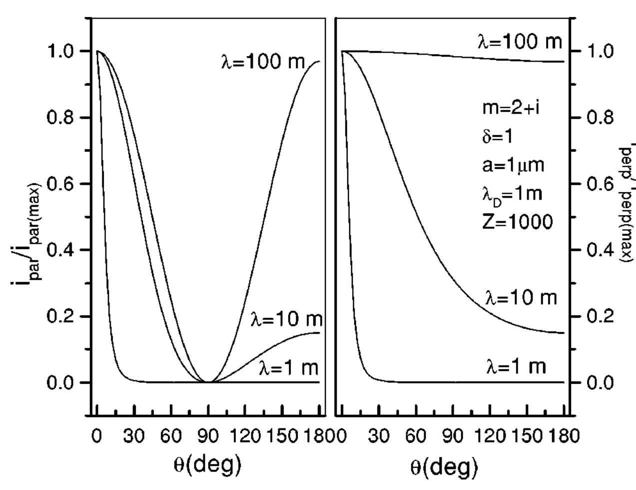


FIG. 3. Radiant intensity normalized by the maximum value at $\theta=0$. The physical parameters are the same as in the preceding figure. The left (right) curves are relative to incident radiation polarized parallel (perpendicular) to the scattering plane (defined incident and scattered wave vectors). For large values of $\lambda$ the curves are equal to the Rayleigh-Mie case and for $\lambda\sim\lambda_D$ the scattering is strongly forward peaked.

In Fig. 4 we show the variation of $d\sigma/d\Omega$ (normalized by $\sigma_0$) with $\lambda$ for $\theta=0$ (forward scattering) and $\theta=\pi$ (backscattering). These curves are calculated from the exact results (4.12) and (4.13). The values used are $a=1$ $\mu$m, $\lambda_D = 7$ mm, and $|Z|=1000$. Thus, they correspond basically to an ionospheric plasma environment.

In both cases the upper curve represents the case $Z>0$, which has a minimum in the backscattering case, but not in the forward scattering. Note that the minimum is attained at different wavelength ranges in each case. In the backscattering it is around 20 mm, almost in the region $\lambda \gg \lambda_D$, whereas in the forward scattering the minimum occurs at 4.8 mm $<\lambda_D$. The reason for this is that the DS is much stronger in the forward than in the backward direction and therefore the wavelength scale on which its magnitude is comparable to that of MS is smaller in the forward than for the backward scattering. Application of expressions (4.14), (4.15) and (4.16) with $k_{0,min}=2\pi/(4.8\times10^{-3})$ m$^{-1}$ and $k_{\pi,min} = 2\pi/(2\times10^{-2})$ m$^{-1}$ allow us to determine $Z/(1+\epsilon)$ and $\lambda_D$.

### *3. Degree of polarization*

Figure 5 shows the polarization degree $P=(I_{\perp}-I_{\parallel})/(I_{\perp}+I_{\parallel})$, calculated through the expressions (4.9) and (4.10). As previously discussed, it is expected that in the Born approximation the polarization curve remains unchanged for all values of the incident wavelength $\lambda$. Numerically, this is observed in this figure, with $a=1$ $\mu$m, $\lambda_D=1$ m, $Z=-1000$ and for three values of $\lambda$, 1, 10, and 100 m. As the value of $\lambda$ decreases more terms in the series (4.9) and (4.10) are needed to get convergence. Nevertheless $P$ tends always to the curve expected.

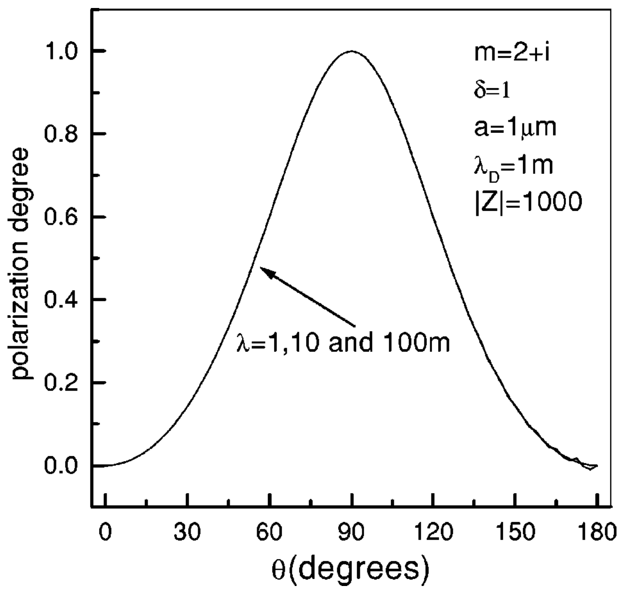


FIG. 5. Polarization degree of the scattered radiation from a depolarized incident field. Plasma parameters identical to those of Fig. 2. The curves are identical for all incident wavelengths, showing the Rayleigh pattern.

## V. A CORRECTION TO THE DISPERSION RELATION FOR A ELECTROMAGNETIC WAVE PROPAGATING IN A PLASMA

In this section we show that the presence of dusts in a plasma introduces a change in the usual dispersion equation, $\omega^2 = \omega_{pe}^2 + c^2k^2$ (where we neglected consistently the ion dynamics).

If we define the (normalized) scattering amplitude $\mathbf{f}(k,\theta,\phi)$ by

$$\mathbf{E}^D(\mathbf{r}) = E_0 \frac{e^{ikr}}{r} \mathbf{f}(k,\theta,\phi), \tag{5.1}$$

then it is a well known result of electrodynamics [27,28] that the dielectric constant of the medium is related to the scattering amplitude in the forward direction by

$$\epsilon(\omega) = 1 + \frac{4\pi N}{k^2} \hat{\boldsymbol{\epsilon}}_0{}^* \cdot \mathbf{f}(k,\theta=0), \tag{5.2}$$

where $N$ is the density of scatterers and $\hat{\boldsymbol{\epsilon}}_0$ is the unit vector in the direction of the incident field. This expression is valid under the conditions of validity of single scattering.

Let us determine what is the influence of the Debye spheres in the dielectric constant. Because the DS is due to the fluctuation in the electron density around the dust, it is the influence of this heterogeneity that we are going to quantify, and not that of the whole bulk of electrons. The latter contributes to the dielectric constant as $-\omega_{pe}^2/\omega^2$, a contribution that we will add to that of the Debye spheres. It should be noted that the contribution of the bulk electrons can be obtained from (5.2) on using $N = n_{eq,e}$ and the scattering amplitude for Thomsom scattering.

We take $N = n_d$, the density of grains, which is related to the electron and ion average densities through (2.1), $\hat{\boldsymbol{\epsilon}}_0 = \hat{\boldsymbol{x}}$ and $\boldsymbol{\epsilon}(k,\theta,\phi)$ is simply given by (4.2) multiplied by $E_0^{-1} r \exp(-ikr)$. To determine $\mathbf{f}(k,\theta=0)$ we use the method in Sec. III A and arrive again at a sum previously used in Sec. IV C, $\Sigma_{n=0}^{\infty}(2n+1)Q_n(1+\epsilon) = 1/\epsilon$. The final result is $\mathbf{f}(k,\theta=0)\cdot\hat{\boldsymbol{x}} = -r_0 Z/(1+\delta)$ and the contribution of the Debye "globules" to the dielectric constant is

$$\epsilon(\omega) = 1 - \frac{\omega_{pe}^2}{k^2c^2} \frac{Zn_d}{n_{eq,e}} \frac{1}{1+\delta}. \tag{5.3}$$

This result was derived from the optical theorem by using the scattering amplitude of transverse electromagnetic waves in a plasma. Therefore, $\epsilon(\omega)$ in (5.3) must be identified with the transverse part, $\epsilon_T(\mathbf{k},\omega)$, of the dielectric tensor $\overline{\overline{\epsilon}}$. The dispersion relation for the transverse electromagnetic wave will be obtained by adding the bulk contribution $-\omega_{pe}^2/\omega^2$ to $\epsilon_T$ and then by writing the usual condition $\epsilon_T(\omega) = (kc/\omega)^2$. This leads to

$$\omega^2 = k^2c^2 + \omega_{pe}^2 \left(1 + \frac{Zn_d}{n_{eq,e}} \frac{1}{1+\delta}\right). \tag{5.4}$$

Therefore, the "globulization" of the dusty plasma introduces a correction in the electron plasma frequency. As expected, it tends to zero as $n_d \to 0$. Simple estimates show that this correction can be of the order of 10%. For example, in a dusty plasma device [29], the following values are possible: $Z \sim 10^3$, $n_{eq,e} \sim 10^8$ cm$^{-3}$ and $n_d \sim 10^4$ cm$^{-3}$. If $\delta \sim 1$, then the correction factor in (5.4) is approximately $\sim 0.1$.

## VI. CONCLUSIONS

We have derived the expression for the field scattered by the Debye cloud around a charged dust in a plasma, given by (3.12). Using the formalism of the Dyadic Green functions we have found the expansion of the Debye field in a set of spherical vector wave functions, in analogy with the Mie field. Thus, the Debye-Mie field can be expressed in a common basis, allowing for a unified treatment. MS dominates at small wavelengths and DS dominates at large wavelengths. In most of the cases there is a minimum in the transition region between these two regimes, where MS and DS are of the same order of magnitude. Three main results were derived from this global view. (1) The total cross section (4.5) has an interference term (4.7) between the Mie and Debye fields, originating a total cross section larger (smaller) than the sum of Debye and Mie individual cross sections, for a positive (negative) grain. This effect can be important in the transition region. (2) The Debye model for the potential allows us to sum exactly the Debye series for forward and backward scattering, leading to simple expressions for the differential cross section in these directions, (4.12) and (4.13). From these expressions a method for determining the values of $Z/(1+\delta)$ and $\lambda_D$ was devised, based on the identification of the minimum of the curves $d\sigma/d\Omega(\theta=0)$ and $d\sigma/d\Omega(\theta=\pi)$. It expressed by (4.14), (4.15), and (4.16). (3) The relation between the dielectric constant and the forward scattering amplitude allows us to derive a correction for the dispersion relation for an electromagnetic wave propagating in a dusty plasma, (5.4). This correction is due to the ''globulization'' of the plasma and can go up to 10% in realistic plasma conditions.

The results for the Debye cross section agree essentially with those of Bingham *et al.* [19] in the long and small wavelength limits, the only cases where a simple analytical comparison is possible. Let us recall that the result of these authors in the high frequency limit ($\omega \gg \omega_{pe}$) is

$$\sigma_D = 24\pi^6 \sigma_0 \ln \frac{2\omega_0}{\omega_p} |Z_k^{\text{eff}}|^2, \tag{6.1}$$

where $Z_\lambda^{\text{eff}}$ is an effective charge, depending on the incident wave number $k$. Their asymptotic expressions are $Z_k^{\text{eff}} \sim Z$ for $\lambda \gg \lambda_D$ and $Z_k^{\text{eff}} \sim Z(\lambda/\lambda_D)^2$ for $\lambda \ll \lambda_D$.

Therefore, the long wavelength limit agrees with (4.8) apart from a numerical factor. The small wavelength limit is in agreement with (4.4).

It should be stressed that our approach is applicable to any model for the potential around the grain. We have chosen the simplest model and yet realistic enough, the standard Debye potential. More complicated models such as those used by Bigham *et al.* [19] or by Whipple *et al.* [5] can be used; it just a matter of substituting (3.2) by the appropriate current term, as long as the linearization of the Maxwell equations can be made, that is $\tilde{n}_e \ll n_{eq,e}$. Of course, analytical results are no longer available, but the numerical calculations can be made.

One more point in favor of the generality of this approach is that the results for the Debye field are applicable even if the dust grain is not spherical, because the Debye cloud still remains approximately spherical. On the contrary, the Mie field depends on the geometry of the particle.

The fact that the dust is treated as a charged sphere raises the question of the influence of the charge in the Mie field. This problem has already been addressed by Bohren and Hunt [30], who have shown that in the case of a dielectric or imperfectly conducting charged sphere there are two contributions for MS, one coming from the bulk dielectric function and the other from surface dielectric functions, associated with the surface currents induced by the incident field. The latter can be included in the Mie theory through a phenomenological surface conductivity, although with limited results and usefulness, mainly because of the difficulties in determining this parameter. Besides, it is expected that this surface contribution give only a small correction to the Mie field. Therefore, in our joint description of Mie and Debye fields we have retained only the dominant bulk contribution for MS.

The Debye sphere acts like a particle, due to coherent scattering of the electrons inside it. Therefore, the Debye scattering should be regarded as scattering by a macroparticle, although it is really the result of scattering by fluctuations in a continuous medium (we have assumed the validity of the fluid model conditions). If there is a collection of randomly distributed dusts, the Born approximation gives incoherent scattering: the total irradiance is the sum of individual irradiances. The dusts can be considered as uncorrelated if the distance between the grains $b$ is larger than $\lambda_D$, $b > \lambda_D$. If this condition is not fulfilled the Debye spheres overlap and our picture of an isolated dressed grain is spoiled. The dust grains become correlated and the total irradiance is much more difficult to calculate.

The application of the proposed method for the determination of the parameters $Z/(1+\delta)$ and $\lambda_D$ is possible for a collection of uncorrelated dust grains, because in this case the relations (4.12) and (4.13) remain valid for the total scattered field from all the scatterers. The case of a collection of correlated dust grains demands further investigation. However, the present results suggest obvious interest in the exploration of the correlated case.

Finally, it is important to mention that very recently astronomic observations have shown that the role of dust emitting in the submillimeter and microwave bands is much more important than previously thought [31]. Presently it is believed that dust emits more radiation in this wavelength range than all the visible stars. Debye scattering can be significant in this region of the electromagnetic spectrum, and thus its study very important for future research in this field.

## APPENDIX: THE SPHERICAL VECTOR WAVE FUNCTIONS

In this appendix a brief summary of the most important formulas related to the set of spherical vector harmonics is presented. Our notation is consistent with Morse and Feschbach [24] and Tai [26].

### 1. Explicit expressions

$$\mathbf{L}_{\sigma mn}(k,\mathbf{r}) = \mathbf{P}_{mn}^{\sigma}(\theta,\phi)\frac{1}{k}\frac{d}{dr}j_n(kr) + \sqrt{n(n+1)} \times \mathbf{B}_{mn}^{\sigma}(\theta,\phi)\frac{1}{kr}j_n(kr). \tag{A1}$$

$$\mathbf{M}_{\sigma mn}(k,\mathbf{r}) = \sqrt{n(n+1)}\,\mathbf{C}_{mn}^{\sigma}(\theta,\phi)j_n(kr), \tag{A2}$$

$$\mathbf{N}_{\sigma mn}(k,\mathbf{r}) = n(n+1)\mathbf{P}_{mn}^{\sigma}(\theta,\phi)\frac{1}{kr}j_n(kr) + \sqrt{n(n+1)} \times \mathbf{B}_{mn}^{\sigma}(\theta,\phi)\frac{1}{kr}\frac{d}{dr}[rj_n(kr)], \tag{A3}$$

where the vector spherical harmonics are

$$\mathbf{B}_{mn}^{\sigma}(\theta,\phi) = \frac{\sqrt{n(n+1)}}{(2n+1)\sin\theta}\left\{\hat{\boldsymbol{\theta}}\left[\frac{n-m+1}{n+1}X_{n+1}^{m}(\theta,\phi) - \frac{n+m}{n}X_{n-1}^{m}(\theta,\phi)\right] + \hat{\boldsymbol{\phi}}\,\frac{m(2n+1)}{n(n+1)}iX_n^m(\theta,\phi)\right\}, \tag{A4}$$

$$C^{\sigma}_{mn}(\theta,\phi)=\frac{\sqrt{n(n+1)}}{(2n+1)\sin\theta}\Bigg\{-\hat{\boldsymbol{\phi}}\Bigg[\frac{n-m+1}{n+1}X^m_{n+1}(\theta,\phi)-\frac{n+m}{n}X^m_{n-1}(\theta,\phi)\Bigg]+\hat{\boldsymbol{\theta}}\frac{m(2n+1)}{n(n+1)}iX^m_n(\theta,\phi)\Bigg\},\tag{A5}$$

$$P^{\sigma}_{mn}(\theta,\phi)=\hat{X}^m_n(\theta,\phi),\tag{A6}$$

where $X^m_n(\theta,\phi)=e^{im\phi}P^m_n(\cos\theta)$.

### 2. Orthogonality relations

$$n(n+1)\int \mathbf{L}_{\sigma mn}(k,\mathbf{r})\cdot\mathbf{L}_{\sigma' m' n'}(k',\mathbf{r})d^3\mathbf{r}=\int \mathbf{M}_{\sigma mn}(k,\mathbf{r})\cdot\mathbf{M}_{\sigma' m' n'}(k',\mathbf{r})d^3\mathbf{r}=\int \mathbf{N}_{\sigma mn}(k,\mathbf{r})\cdot\mathbf{N}_{\sigma' m' n'}(k',\mathbf{r})d^3\mathbf{r}=\frac{2\pi^2}{h^2C_{mn}}\delta_{\sigma\sigma'}\delta_{mm'}\delta_{nn'}\delta(k-k'),\tag{A7}$$

where $C_{mn}=(2-\delta_0)(2n+1)(n-m)!/n(n+1)(n+m)!$,

$$\int B^{\sigma}_{mn}(\theta,\phi)B^{\sigma'}_{m'n'}(\theta,\phi)d\Omega=\int C^{\sigma}_{mn}(\theta,\phi)C^{\sigma'}_{m'n'}(\theta,\phi)d\Omega=\int P^{\sigma}_{mn}(\theta,\phi)P^{\sigma'}_{m'n'}(\theta,\phi)d\Omega=\frac{4\pi/\epsilon_m}{2n+1}\frac{(n+m)!}{(n-m)!}\delta_{\sigma\sigma'}\delta_{mm'}\delta_{nn'},\tag{A8}$$

with $\epsilon_m=1$ if $m=0$ and $=2$ if $m\neq 0$.


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